



An Algorithmic Approach to Compute KCD Indices of Generalized Transformation Graphs G^{yz} and their Complements.

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Abstract

This article focuses on the study of KCD indices for generalized transformation graphs G^{yz} and their complements. In this study, the expressions for KCD indices of G^{yz} and $\overline{G^{yz}}$ are obtained. Further the results are verified by an algorithmic approach.

Keywords: KCD indices, Generalized transformation graphs, Algorithm.

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1. Introduction

The graph G considered in this article is simple, connected and finite of order n and size m . The vertex degree $\deg_G(v)$ is the sum of all the edges incident to the vertex $v \in V(G)$. The complement graph \overline{G} with $v \in V(G)$ is a graph having two vertices as adjacent if they are non-adjacent in G . The notations and terminologies which are not defined are cited from [5].

A numerical quantity computed for a graph which is obtained from the molecular graph is called as topological index. Topological indices are used to analyse mathematical values. Extensive research work on topological index with respect to vertex degree has been documented in [2]. H. Wiener introduced the topological index with respect to distance in 1947 [12]. Randić proposed the first vertex degree based topological index in 1975 known as connectivity index or Randić index [10]. Zagreb indices were defined in 1972 [3], where the first Zagreb index is

$$M_1(G) = \sum_{e=uv \in E(G)} \left(\deg_G(u) + \deg_G(v) \right).$$

Voluminous research work has been published on various topological indices by several mathematicians [4, 6, 7, 11].

One of the fascinating branch in graph theory is graph transformation which has received the most attention in the research field. The technique of obtaining a new graph from the given graph by making

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a few changes in the given graph is termed as graph transformation. In particular, this technique uses the incidence or non-incidence relationship between vertices and edges along with the adjacency or non-adjacency relationship between two vertices, two edges, two cutvertices and so on is known as graph transformation.

Algorithms are the instructions for solving a problem or completing a task. An algorithmic approach will make the computation process more efficient and easier with accuracy. Developing an algorithm enables computation process in logical manner. Hence in this article effort has been made to design an algorithm to compute KCD indices for G^{yz} and $\overline{G^{yz}}$.

2. Preliminaries

Generalized Transformation Graph G^{yz} [13, 1] is a graph having a vertex set $V(G) \cup E(G)$ and $\eta, \zeta \in V(G^{yz})$, where y and z are variables take the values + or -. The vertices η and ζ are adjacent in G^{yz} if and only if (a) and (b) holds:

(a) For $\eta, \zeta \in V(G)$ and if η and ζ are adjacent in G then $y = +$ otherwise $y = -$. Here $y = +$ and $y = -$ represent the adjacency relationship.

(b) For $\eta \in V(G)$ and $\zeta \in E(G)$ and if η and ζ are incident in G , then $z = +$ otherwise $z = -$. Here $z = +$ and $z = -$ represent the incidence relationship.

There are four graphical transformations of a graph, such as G^{++} , G^{+-} , G^{-+} and G^{--} . Let $\overline{G^{yz}}$ is the complement of transformation graph G^{yz} .

Recently a set of vertex-edge degree based topological indices are presented in [8] and termed as KCD (Karnatak College Dharwad) indices.

The first and second KCD indices are

$$KCD_1(G) = \sum_{e=uv \in E(G)} \left((\deg_G(u) + \deg_G(v)) + \deg_G(e) \right) \tag{2.1}$$

$$KCD_2(G) = \sum_{e=uv \in E(G)} \left((\deg_G(u) + \deg_G(v)) \deg_G(e) \right), \tag{2.2}$$

where $\deg_G(u)$ and $\deg_G(v)$ represent the vertex degree and $\deg_G(e) = \deg_G(u) + \deg_G(v) - 2$ represents the edge degree.

Below mentioned propositions are of immediate use in proof of the results.

Proposition 2.1. [9] For G with $u \in V(G)$, $e \in E(G)$

- (a) $\deg_{G^{++}}(u) = 2\deg_G(u)$ and $\deg_{G^{++}}(e) = 2$.
- (b) $\deg_{G^{+-}}(u) = m$ and $\deg_{G^{+-}}(e) = n - 2$.
- (c) $\deg_{G^{-+}}(u) = n - 1$ and $\deg_{G^{-+}}(e) = 2$.
- (d) $\deg_{G^{--}}(u) = n + m - 1 - 2\deg_G(u)$ and $\deg_{G^{--}}(e) = n - 2$.

Proposition 2.2. [9] For G with $u \in V(G)$, $e \in E(G)$

- (a) $E(G^{++})$ is divided into sets E_1 and E_2 , $E_1 = \{uv | uv \in E(G)\}$ and $E_2 = \{ue | u \text{ is incident to } e \text{ in } G\}$, where $|E_1| = m$ and $|E_2| = 2m$.
- (b) $E(G^{+-})$ is divided into sets E_1 and E_2 , $E_1 = \{uv | uv \in E(G)\}$ and $E_2 = \{ue | u \text{ is not incident to } e \text{ in } G\}$, where $|E_1| = m$ and $|E_2| = m(n - 2)$.
- (c) $E(G^{-+})$ is divided into sets E_1 and E_2 , $E_1 = \{uv | uv \notin E(G)\}$ and $E_2 = \{ue | u \text{ is incident to } e \text{ in } G\}$, where $|E_1| = \binom{n}{2} - m$ and $|E_2| = 2m$.
- (d) $E(G^{--})$ is divided into sets E_1 and E_2 , $E_1 = \{uv | uv \notin E(G)\}$ and $E_2 = \{ue | u \text{ is not incident to } e \text{ in } G\}$, where $|E_1| = \binom{n}{2} - m$ and $|E_2| = m(n - 2)$.

Proposition 2.3. [9] For G with $u \in V(G)$, $e \in E(G)$

- (a) $\deg_{\overline{G^{++}}}(u) = n + m - 1 - 2\deg_G(u)$ and $\deg_{\overline{G^{++}}}(e) = n + m - 3$.
- (b) $\deg_{\overline{G^{+-}}}(u) = n - 1$ and $\deg_{\overline{G^{+-}}}(e) = m + 1$.
- (c) $\deg_{\overline{G^{-+}}}(u) = m$ and $\deg_{\overline{G^{-+}}}(e) = n + m - 3$.

(d) $\deg_{\overline{G}^{--}}(u) = 2\deg_G(u)$ and $\deg_{\overline{G}^{--}}(e) = m + 1$.

Proposition 2.4. [9] For G with $u \in V(G)$, $e \in E(G)$

(a) $E(\overline{G}^{++})$ is divided into sets E_1, E_2 and E_3 , $E_1 = \{uv | uv \notin E(G)\}$, $E_2 = \{ue | u \text{ is not incident to } e \text{ in } G\}$ and $E_3 = \{ef | e, f \in E(G)\}$, where $|E_1| = \binom{n}{2} - m$, $|E_2| = m(n - 2)$ and $E_3 = \binom{m}{2}$.

(b) $E(\overline{G}^{+-})$ is divided into sets E_1, E_2 and E_3 , $E_1 = \{uv | uv \notin E(G)\}$, $E_2 = \{ue | u \text{ is incident to } e \text{ in } G\}$ and $E_3 = \{ef | e, f \in E(G)\}$, where $|E_1| = \binom{n}{2} - m$, $|E_2| = 2m$ and $E_3 = \binom{m}{2}$.

(c) $E(\overline{G}^{-+})$ is divided into sets E_1, E_2 and E_3 , $E_1 = \{uv | uv \in E(G)\}$, $E_2 = \{ue | u \text{ is not incident to } e \text{ in } G\}$ and $E_3 = \{ef | e, f \in E(G)\}$, where $|E_1| = m$, $|E_2| = 2m(n - 2)$ and $E_3 = \binom{m}{2}$.

(d) $E(\overline{G}^{--})$ is divided into sets E_1, E_2 and E_3 , $E_1 = \{uv | uv \in E(G)\}$, $E_2 = \{ue | u \text{ is incident to } e \text{ in } G\}$ and $E_3 = \{ef | e, f \in E(G)\}$, where $|E_1| = m$, $|E_2| = 2m$ and $E_3 = \binom{m}{2}$.

In this paper the results on generalized transformation graphs and their complements for first and second KCD indices are obtained. An algorithm is proposed to verify the results.

3. Results

3.1. First and second KCD indices of G^{yz}

Theorem 3.1.1 For any graph G ,

$$KCD_1(G^{++}) = 4M_1(G) - 2m + \sum_{ue \in E_2} 2 \left(2\deg_G(u) + 1 \right).$$

Proof. From Equation (2.1), we get

$$\begin{aligned} KCD_1(G^{++}) &= \sum_{uv \in E(G^{++})} \left(2\deg_{G^{++}}(u) + 2\deg_{G^{++}}(v) - 2 \right) \\ &= \sum_{uv \in E_1} \left(2\deg_{G^{++}}(u) + 2\deg_{G^{++}}(v) - 2 \right) + \sum_{ue \in E_2} \left(2\deg_{G^{++}}(u) + 2\deg_{G^{++}}(e) - 2 \right) \\ &= \sum_{uv \in E(G)} \left(2(2\deg_G(u)) + 2(2\deg_G(v)) - 2 \right) \\ &\quad + \sum_{ue \in E_2} \left(2(2\deg_G(u)) + 4 - 2 \right) \quad \text{using Proposition 2.1(a)} \\ &= \sum_{uv \in E(G)} 2 \left(2\deg_G(u) + 2\deg_G(v) - 1 \right) + \sum_{ue \in E_2} 2 \left(2\deg_G(u) + 1 \right) \\ &= 4M_1(G) - 2m + \sum_{ue \in E_2} 2 \left(2\deg_G(u) + 1 \right). \quad \text{using Proposition 2.2(a)} \end{aligned}$$

Theorem 3.1.2 For any graph G ,

$$\begin{aligned} KCD_2(G^{++}) &= \sum_{uv \in E(G)} 4 \left(\deg_G(u) + \deg_G(v) \right) \left(\deg_G(u) + \deg_G(v) - 1 \right) \\ &\quad + \sum_{ue \in E_2} 4\deg_G(u) \left(\deg_G(u) + 1 \right). \end{aligned}$$

Proof. Proof follows from Equation (2.2) and Propositions 2.1 (a).

Theorem 3.1.3 For any graph G ,

$$\text{KCD}_1(G^{+-}) = 2m(n^2 + nm - 5n + 5).$$

Proof. From Equation (2.1), we get

$$\begin{aligned} \text{KCD}_1(G^{+-}) &= \sum_{uv \in E(G^{+-})} \left(2\text{deg}_{G^{+-}}(u) + 2\text{deg}_{G^{+-}}(v) - 2 \right) \\ &= \sum_{uv \in E_1} \left(2\text{deg}_{G^{+-}}(u) + 2\text{deg}_{G^{+-}}(v) - 2 \right) + \sum_{ue \in E_2} \left(2\text{deg}_{G^{+-}}(u) + 2\text{deg}_{G^{+-}}(e) - 2 \right) \\ &= \sum_{uv \in E(G)} \left(4m - 2 \right) + \sum_{ue \in E_2} \left(2m + 2n - 6 \right) && \text{using Proposition 2.1(b)} \\ &= 2m(n^2 + nm - 5n + 5). && \text{using Proposition 2.2(b)} \end{aligned}$$

Theorem 3.1.4 For any graph G ,

$$\text{KCD}_2(G^{+-}) = 4m(m - 1) + m(n - 2)(m^2 + n^2 - 2nm - 6n - 6m - 8).$$

Proof. Proof follows from Equation (2.2) and Propositions 2.1(b) and 2.2(b).

Theorem 3.1.5 For any graph G ,

$$\text{KCD}_1(G^{-+}) = (2n - 3)(n^2 - n - 2m) + 4nm.$$

Proof. From Equation (2.1), we get

$$\begin{aligned} \text{KCD}_1(G^{-+}) &= \sum_{uv \in E(G^{+-})} \left(2\text{deg}_{G^{-+}}(u) + 2\text{deg}_{G^{-+}}(v) - 2 \right) \\ &= \sum_{uv \in E_1} \left(2\text{deg}_{G^{-+}}(u) + 2\text{deg}_{G^{-+}}(v) - 2 \right) + \sum_{ue \in E_2} \left(2\text{deg}_{G^{-+}}(u) + 2\text{deg}_{G^{-+}}(e) - 2 \right) \\ &= \sum_{uv \notin E(G)} (4n - 6) + \sum_{ue \in E_2} 2n && \text{using Proposition 2.1(c)} \\ &= (2n - 3)(n^2 - n - 2m) + 4nm. && \text{using Proposition 2.2(c)} \end{aligned}$$

Theorem 3.1.6 For any graph G ,

$$\text{KCD}_2(G^{-+}) = 2 \left((n^2 - 3n + 2)(n^2 - n - 2m) + m(n^2 - 1) \right).$$

Proof. Proof follows from Equation (2.2) and Propositions 2.1(c) and 2.2(c).

Theorem 3.1.7 For any graph G ,

$$\begin{aligned} \text{KCD}_1(G^{--}) &= \sum_{uv \notin E(G)} 2 \left(2n + 2m - 3 - 2\text{deg}_G(u) - 2\text{deg}_G(v) \right) \\ &\quad + \sum_{ue \in E_2} 2 \left(2n + m - 4 - 2\text{deg}_G(u) \right). \end{aligned}$$

Proof. From Equation (2.1), we get

$$\begin{aligned} \text{KCD}_1(G^{--}) &= \sum_{uv \in E(G^{--})} \left(2\text{deg}_{G^{--}}(u) + 2\text{deg}_{G^{--}}(v) - 2 \right) \\ &= \sum_{uv \in E_1} \left(2\text{deg}_{G^{--}}(u) + 2\text{deg}_{G^{--}}(v) - 2 \right) + \sum_{ue \in E_2} \left(2\text{deg}_{G^{--}}(u) + 2\text{deg}_{G^{--}}(e) - 2 \right) \\ &= \sum_{uv \notin E(G)} 2 \left(2n + 2m - 3 - 2\text{deg}_G(u) - 2\text{deg}_G(v) \right) \\ &\quad + \sum_{ue \in E_2} 2 \left(2n + m - 4 - 2\text{deg}_G(u) \right). \quad \text{using Proposition 2.1(d)} \end{aligned}$$

Theorem 3.1.8 For any graph G ,

$$\begin{aligned} \text{KCD}_2(G^{--}) &= \sum_{uv \in E_1} \left(2n + 2m - 2 - 2\text{deg}_G(u) - 2\text{deg}_G(v) \right) \left(2n + 2m - 4 - 2\text{deg}_G(u) - 2\text{deg}_G(v) \right) \\ &\quad + \sum_{ue \in E_2} \left(2n + m - 3 - 2\text{deg}_G(u) \right) \left(2n + m - 5 - 2\text{deg}_G(u) \right). \end{aligned}$$

Proof. Proof follows from Equation (2.2) and Propositions 2.1(d).

3.2. First and second KCD indices of $\overline{G^{xy}}$

Theorem 3.2.1 For any graph G ,

$$\begin{aligned} \text{KCD}_1(\overline{G^{++}}) &= \sum_{uv \in E(G)} \left(4n + 4m - 6 - 4\text{deg}_G(u) - 4\text{deg}_G(v) \right) \\ &\quad + \sum_{ue \in E_2} \left(4n + 4m - 10 - 4\text{deg}_G(u) \right) \\ &\quad + m(m-1)(2n + 2m - 7). \end{aligned}$$

Proof. From Equation (2.1), we get

$$\begin{aligned} \text{KCD}_1(\overline{G^{++}}) &= \sum_{uv \in E(\overline{G^{++}})} \left(2\text{deg}_{\overline{G^{++}}}(u) + 2\text{deg}_{\overline{G^{++}}}(v) - 2 \right) \\ &= \sum_{uv \in E_1} \left(2\text{deg}_{\overline{G^{++}}}(u) + 2\text{deg}_{\overline{G^{++}}}(v) - 2 \right) + \sum_{ue \in E_2} \left(2\text{deg}_{\overline{G^{++}}}(u) + 2\text{deg}_{\overline{G^{++}}}(e) - 2 \right) \\ &\quad + \sum_{ef \in E_3} \left(2\text{deg}_{\overline{G^{++}}}(e) + 2\text{deg}_{\overline{G^{++}}}(f) - 2 \right) \\ &= \sum_{uv \in E(G)} \left(2(n + m - 1 - 2\text{deg}_G(u)) + 2(n + m - 1 - 2\text{deg}_G(v)) - 2 \right) \\ &\quad + \sum_{ue \in E_2} \left(2(n + m - 1 - 2\text{deg}_G(u)) + 2(n + m - 3) - 2 \right) \\ &\quad + \sum_{ef \in E_3} \left(2(n + m - 3) + 2(n + m - 3) - 2 \right) \quad \text{using Proposition 2.3(a)} \end{aligned}$$

$$\begin{aligned}
 &= \sum_{uv \in E(G)} (4n + 4m - 6 - 4\deg_G(u) - 4\deg_G(v)) \\
 &+ \sum_{ue \in E_2} (4n + 4m - 5 - 4\deg_G(u)) \\
 &+ \sum_{ef \in E_3} (4n + 4m - 14) \\
 &= \sum_{uv \in E(G)} (4n + 4m - 6 - 4\deg_G(u) - 4\deg_G(v)) \\
 &+ \sum_{ue \in E_2} (4n + 4m - 5 - 4\deg_G(u)) \\
 &+ \frac{m(m-1)}{2}(4n + 4m - 14) \qquad \text{using Proposition 2.4(a)} \\
 &= \sum_{uv \in E(G)} (4n + 4m - 6 - 4\deg_G(u) - 4\deg_G(v)) \\
 &+ \sum_{ue \in E_2} (4n + 4m - 10 - 4\deg_G(u)) \\
 &+ m(m-1)(2n + 2m - 7).
 \end{aligned}$$

Theorem 3.2.2 For any graph G ,

$$\begin{aligned}
 \text{KCD}_2(\overline{G^{++}}) &= \sum_{uv \in E_1} 4 \left(n + m - 1 - \deg_G(u) - \deg_G(v) \right) \left(n + m - 2 - \deg_G(u) - \deg_G(v) \right) \\
 &+ \sum_{ue \in E_2} 4 \left(n + m - 2 - \deg_G(u) \right) \left(n + m - 3 - \deg_G(u) \right) \\
 &+ 2m(m-1)(n^2 + m^2 + 2nm - 7n - 7m + 12).
 \end{aligned}$$

Proof. Proof follows from Equation (2.2) and Propositions 2.3(a) and 2.4(a).

Theorem 3.2.3 For any graph G ,

$$\text{KCD}_1(\overline{G^{+-}}) = (n^2 - n - 2m)(2n - 3) + m \left(4(n + m - 1) + (m - 1)(2m + 1) \right).$$

Proof. From Equation (2.1), we get

$$\begin{aligned}
 \text{KCD}_1(\overline{G^{+-}}) &= \sum_{uv \in E(\overline{G^{+-}})} \left(2\deg_{\overline{G^{+-}}}(u) + 2\deg_{\overline{G^{+-}}}(v) - 2 \right) \\
 &= \sum_{uv \in E_1} \left(2\deg_{\overline{G^{+-}}}(u) + 2\deg_{\overline{G^{+-}}}(v) - 2 \right) + \sum_{ue \in E_2} \left(2\deg_{\overline{G^{+-}}}(u) + 2\deg_{\overline{G^{+-}}}(e) - 2 \right) \\
 &+ \sum_{ef \in E_3} \left(2\deg_{\overline{G^{+-}}}(e) + 2\deg_{\overline{G^{+-}}}(f) - 2 \right) \\
 &= \sum_{uv \in E(G)} (4n - 6) + \sum_{ue \in E_2} (2n + 2m - 2) + \sum_{ef \in E_3} (4m + 2) \qquad \text{using Proposition 2.3(b)} \\
 &= (n^2 - n - 2m)(2n - 3) + m(4(n + m - 1) + (m - 1)(2m + 1)). \qquad \text{using Proposition 2.4(b)}
 \end{aligned}$$

Theorem 3.2.4 For any graph G ,

$$\text{KCD}_2(\overline{G^{+-}}) = 2\left(n^2 - 3n + 2\right)\left(n^2 - n - 2m\right) + 2m\left((n + m)(n + m - 2) + m(m^2 - 1)\right).$$

Proof. Proof follows from Equation (2.2) and Propositions 2.3(b) and 2.4(b).

Theorem 3.2.5 For any graph G ,

$$\text{KCD}_1(\overline{G^{-+}}) = 2m(2m - 1) + m(2(n - 2)(2m + n - 4) + (m - 1)(2n + 2m - 7)).$$

Proof. From Equation (2.1), we get

$$\begin{aligned} \text{KCD}_1(\overline{G^{-+}}) &= \sum_{uv \in E(\overline{G^{-+}})} \left(2\text{deg}_{\overline{G^{-+}}}(u) + 2\text{deg}_{\overline{G^{-+}}}(v) - 2\right) \\ &= \sum_{uv \in E_1} \left(2\text{deg}_{\overline{G^{-+}}}(u) + 2\text{deg}_{\overline{G^{-+}}}(v) - 2\right) + \sum_{ue \in E_2} \left(2\text{deg}_{\overline{G^{-+}}}(u) + 2\text{deg}_{\overline{G^{-+}}}(e) - 2\right) \\ &\quad + \sum_{ef \in E_3} \left(2\text{deg}_{\overline{G^{-+}}}(e) + 2\text{deg}_{\overline{G^{-+}}}(f) - 2\right) \\ &= \sum_{uv \in E(G)} (4m - 2) + \sum_{ue \in E_2} (4m + 2n - 8) \\ &\quad + \sum_{ef \in E_3} (4n + 4m - 14) \qquad \text{using Proposition 2.3(c)} \\ &= 2m\left(2m - 1\right) + m\left(2(n - 2)(2m + n - 4)\right. \\ &\quad \left.+ (m - 1)(2n + 2m - 7)\right). \qquad \text{using Proposition 2.4(c)} \end{aligned}$$

Theorem 3.2.6 For any graph G ,

$$\text{KCD}_2(\overline{G^{++}}) = 2m(m - 1)\left(2m + (n + m - 3)(n + m - 4)\right) + m(m - 2)\left(2m + n - 3\right)\left(2m + n - 5\right).$$

Proof. Proof follows from Equation (2.2) and Propositions 2.3(c) and 2.4(c).

Theorem 3.2.7 For any graph G ,

$$\text{KCD}_1(\overline{G^{--}}) = \sum_{uv \in E(G)} 2\left(2\text{deg}_G(u) + 2\text{deg}_G(v) - 1\right) + \sum_{ue \in E_2} 2\left(2\text{deg}_G(u) + m\right) + m\left(m^2 - m - 1\right).$$

Proof. From Equation (2.1) and Propositions 2.3 and 2.4, we get

$$\begin{aligned}
 \text{KCD}_1(\overline{G^{--}}) &= \sum_{uv \in E(\overline{G^{--}})} \left(2\text{deg}_{\overline{G^{--}}}(u) + 2\text{deg}_{\overline{G^{--}}}(v) - 2 \right) \\
 &= \sum_{uv \in E_1} \left(2\text{deg}_{\overline{G^{--}}}(u) + 2\text{deg}_{\overline{G^{--}}}(v) - 2 \right) + \sum_{ue \in E_2} \left(2\text{deg}_{\overline{G^{--}}}(u) + 2\text{deg}_{\overline{G^{--}}}(e) - 2 \right) \\
 &\quad + \sum_{ef \in E_3} \left(2\text{deg}_{\overline{G^{--}}}(e) + 2\text{deg}_{\overline{G^{--}}}(f) - 2 \right) \\
 &= \sum_{uv \in E(G)} \left(4\text{deg}_G(u) + 4\text{deg}_G(v) - 2 \right) + \sum_{ue \in E_2} \left(4\text{deg}_G(u) + 2(m+1) - 2 \right) \\
 &\quad + \sum_{ef \in E_3} (4m+2) \qquad \text{using Proposition 2.3(d)} \\
 &= \sum_{uv \in E(G)} 2 \left(2\text{deg}_G(u) + 2\text{deg}_G(v) - 1 \right) \\
 &\quad + \sum_{ue \in E_2} 2 \left(2\text{deg}_G(u) + m \right) + m \left(m^2 - m - 1 \right). \qquad \text{using Proposition 2.4(d)}
 \end{aligned}$$

Theorem 3.2.8 For any graph G ,

$$\begin{aligned}
 \text{KCD}_2(\overline{G^{--}}) &= \sum_{uv \in E(G)} 4 \left(\text{deg}_G(u) + \text{deg}_G(v) \right) \left(\text{deg}_G(u) + \text{deg}_G(v) - 1 \right) \\
 &\quad + \sum_{ue \in E_2} \left((2\text{deg}_G(u) + m)^2 - 1 \right) + 2m^2 \left(m^2 - 1 \right).
 \end{aligned}$$

Proof. Proof follows from Equation (2.2) and Propositions 2.3(d) and 2.4(d).

4. Algorithm to compute KCD indices

In this section, an algorithm is designed to make the computation of KCD indices convenient and ease for any kind of connected graph.

Step 1: START

Step 2: [Reading values]

Read n , graph-type.

Step 4: [Checking wheather graph-type is directed or undirected]

if(graph-type is undirected)

begin

$\text{maxedges} \leftarrow n * (n - 1) / 2$

end

else

begin

$\text{maxedges} \leftarrow n * (n - 1)$

end

Step 3: [Creating an adjacency matrix]

for i to maxedges

begin

Read ori , dest


```

    if ((ori == 0) and (dest == 0))
        break
    if (ori > n or dest > n or ori ≤ 0 or dest ≤ 0)
        begin
            Display "Invalid edge"
        end
    else
        begin
            adj[ori][dest] ← 1
        end
    if(graph-type==undirected)
        begin
            adj[dest][ori] ← 1
        end
    end
end

```

Step 5: [Computing the degree of every vertex]

```

for i = 1 to n
    begin
        deg[i] ← 0
        for j to n
            begin
                deg[i] ← deg[i]+a[i][j]
            end
        end
        Display deg[i]
    end
end

```

Step 6: [Computing first and second KCD indices]

```

for i = 1 to n
    begin
        for j = 1 to n
            if (a[i][j] == 1 and i < j)
                begin
                    KCD1 ← KCD1 + 2 * deg[i] + 2 * deg[j] - 2
                    KCD2 ← KCD2 + (deg[i] + deg[j]) * (deg[i] + deg[j] - 2)
                end
            end
        end
    end
end
end

```

Step 7: [Displaying the result]

```

Display KCD1, KCD2

```

Step 8: STOP.

This algorithm reads input as adjacent edges and graph type. It checks whether the graph type is directed or undirected and accordingly creates adjacency matrix $adj[i][j]$. Further it computes the degree of each vertex and correspondingly computes KCD1 and KCD2 for adjacent vertices.

As adjacency matrix $adj[i][j]$ is stored using 2-dimensional array, the outer loop and inner loop executes based on the array size n . The execution time required to complete both loops is $n * n$ times. Thus time complexity of this algorithm is $O(n^2)$.

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